Q6

(a) $f(x) = 1 + 3x^2 - 2x^3$

$f'(x) = 0$

$\frac{\partial}{\partial x}(1 + 3x^2 - 2x^3) = 0$

$6x - 6x^2 = 0 \Rightarrow 6x(1-x) = 0$

$x = 0, x = 1$

By first derrvative test : $f'(x) > 0$ when $0 < x < 1$
and $f'(x) < 0$ when $x < 0$ or $x > 1$
f increasing on the interval $(0, 1)$ hence f has
maximum value at $x = 1$
f decreasing on $(-\infty, 0)$ and increasing $1 > 0$
f has has minimum value at $x = 0$.

(b) $f(x) = \dfrac{x^2}{x-1}$

Critical points at $f'(x) = 0$.

$$f'(x) = \frac{\partial}{\partial x}\left(\frac{x^2}{x-1}\right) = 0.$$

$$\frac{\partial}{\partial x}\left(\frac{x^2}{x-1}\right) = \frac{\frac{\partial}{\partial x}(x^2)(x-1) - \frac{\partial}{\partial x}(x-1)x^2}{(x-1)x} = 0.$$

$$= \frac{2x(x-1) - 1 \cdot x^2}{(x-1)^2} = \frac{x^2-2x}{(x-1)^2} = 0$$

$x^2 - 2x = 0$, $x^2 = 2x$, $x = 2$.

$x(x-2) = 0$, $x = 0$ ~~$x-1=0$~~, $x = 0$.

By first derivative test $f'(x) > 0$ when $x > 2$

$f'(x) < 0$ when $x < 0$

$f'x > 0$ whe $0 < x < 2$

$f$ is increasing on $(-\infty, 0)$ and then decreasing hence

$f$ is maximum at $x = 0$, $f$ is decreasing at $(0, 2)$

$f$ is minimum at $x = 2$.

$2^{nd}$ derivative : $f''(x) = \frac{\partial}{\partial x}\left(\frac{x^2 - 2x}{(x-1)^2}\right)$

$$\frac{\partial}{\partial x}\left(\frac{x^2-2x}{(x-1)^2}\right) = \frac{\frac{\partial}{\partial x}(x^2-2x)(x-1)^2 - \frac{\partial}{\partial x}((x-1)^2)(x^2-2x)}{((x-1)^2)^2}$$

$$= \frac{(2x-2)(x-1)^2 - 2(x-1)(x^2-2x)}{((x-1)^2)^2}$$

$$= \frac{(2x-2)(x-1)^2 - 2(x-1)(x^2-2x)}{((x-1)^2(x-1)^2} = \frac{2}{(x-1)^3}$$

$$f''(x) = \frac{2}{(x-1)^3} \neq 0.$$

$$f''(0) = \frac{2}{(-1)^3} = \frac{2}{-1} < 0$$

$f$ is maximum at 0, $f''(2) = \frac{2}{-1} = -2 < 0$

for minimum at $x = 2$

$$f(0) = \frac{0}{0-1} = 0, \quad f(2) = \frac{(2)^2}{2-1} = \frac{4}{1} = 4.$$

~~Local~~ maximum value = 0 at $x = 0$

minimum value = 4 at $x = 2$.

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